Overlap domain decomposition method for wave propagation

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SUMMARY

A computationally efficient overlap domain decomposition method (ODD) for modeling wave propagation in large domains is presented. The ODD method divides a large domain into several small overlapping subdomains. The method allows the exchange of waves between the subdomains through the overlapping regions without the need for any internal interface connecting conditions. For each subdomain, calculations are performed independently. Thus, it is possible to use different numerical algorithms in each subdomain. The method not only saves computing time, but also gives nearly the same accuracy as conventional methods. This paper describes the ODD algorithm when wave propagation in the subdomains are computed using the finite difference and the pseudospectral methods. An example of the method for a two-dimensional domain divided into 25 subdomains demonstrates the computing efficiency and flexibility of the ODD method.

INTRODUCTION

Recent work has shown that significant advantages can be obtained by using domain decomposition methods to numerically simulate wave propagation. Domain decomposition methods are suitable for parallel computing and, in addition, allow different numerical methods to be coupled together. To date, most of these approaches have utilized non-overlapping subdomains (Tessmer, et al., 1992; Carcione, 1991; Faccioli et al., 1996). A potential limitation of non-overlapping methods are the need for interface conditions at the (artificial) subdomain boundaries. Overlap domain decomposition (ODD) methods for parabolic equations have been proposed by Kuznetsov (1988) and Chen and Lazarov (1994) for finite element and finite difference methods. Liao and McMechan (1993) applied ODD to Fourier pseudospectral method for viscoacoustic modeling. Israeli et al. (1994) develped ODD methods with local Fourier basis for parabolic problems.

We present a noniterative overlap domain decomposition method for wave propagation problems based on Huygen's Principle. A large domain is split into several small overlapping subdomains. Waves are passed from one subdomain to adjacent subdomains by the overlap regions. The wavefield is independently calculated in each subdomain by using a noniterative explicit algorithm. The wavefield in the whole domain is obtained from the local solutions in the subdomains. Different subdomains can adopt different methods such as finite difference, pseudospectral, and other methods.

The advantages of this ODD method are: (1) parallel computing techniques can be easily applied since each subdomain is computed independently of its neighbors; (2) different subdomains can use different methods; (3) computing time and memory can be reduced by decreasing the computing domain size and by turning-off the calculations in the subdomains which do not have wave activity.

The ODD methods is described for a one-dimensional (1D) medium and implemented for finite difference (FD), pseudospectral (PS) and mixed FD-PS methods. Examples of method are given for both 1-D and 2-D cases.

ODD METHOD FOR 1-D PROBLEMS

The ODD method is described schematically in Fig. 1 for one dimensional wave propagation. The method can be described by the following steps:

Step 1. A large domain Ω [a, e] containing a wavefield $f(x,n)$ is split into two subdomains $\Omega 1$ [a, d] with the function $f1(x,n)$ and $\Omega 2$ [b, e] with the function $f2(x,n)$ at time step n.

$$f1(x,n) = f(x,n) \quad x \in [a, d] \text{ in } \Omega 1$$
$$f2(x,n) = f(x,n) \quad x \in [b, e] \text{ in } \Omega 2$$

Subdomains $\Omega 1$ and $\Omega 2$ have a common region Ωo [b, d] from the original domain Ω which is called the *overlap region*. Both subdomains carry a common part of the wave. At the same time, the domain splitting introduces two artificial boundaries in the overlap region Ωo [b, d] at point d for $\Omega 1$ and at point b for Ω.

Step 2. The solution at a time step n+1 in $\Omega 1$ and $\Omega 2$ can be calculated independently in $\Omega 1$ and $\Omega 2$. The overlap region Ωo [b, d] belongs to both subdomains and the wavefield in Ωo is independently calculated twice. Although the artificial boundaries generate reflected waves, according to Huygen's Principle, during the small time step *dt* the reflected waves should only affect a small area of the dimension *dx=velocity*dt* near the artificial boundaries. If the length of the overlap region Ωo is chosen longer than *2dx*, then the reflected waves only occur within the half overlap regions $\Omega o2$ [c, d] in $\Omega 1$ and $\Omega o1$ [b, c] in $\Omega 2$ near the boundaries. The wavefield is not affected within the half overlap regions

Ωo1 [b, c] in Ω1 and Ωo2 [c, d] in Ω2 by these artificial reflections.

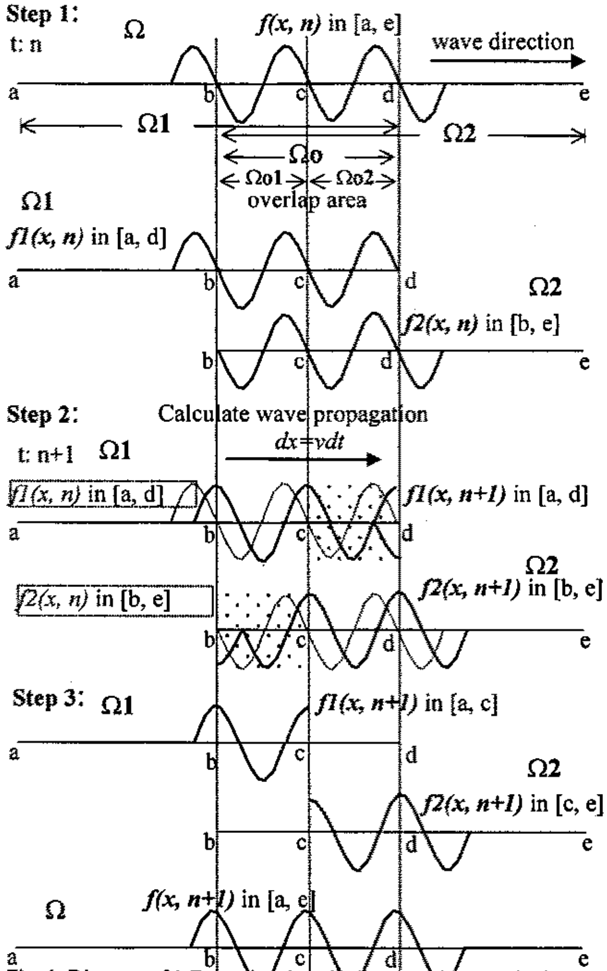

Fig. 1 Diagram of 1-D overlap domain decomposition method

Step 3. Take the wavefield from the region [a, c] in Ω1 and the region [c, e] in Ω2 to form the whole domain wavefield.

$$f(x,n+1)=\begin{cases} f1(x,n+1) & x\in[a,\ c] \\ f2(x,n+1) & x\in[c,\ e] \end{cases}$$

The reflected waves from the artificial boundaries are eliminated by only taking the contributions from the respective solutions corresponding to the half of the overlap region within each subdomain. The choice of size of the overlap region depends on the numerical method used for wave propagation.

ODD for 1-D finite difference (FD) method

The 1-D finite difference method can be directly implemented into the ODD method. Starting with the 1-D wave equation for a medium with constant density ρ,

$$\frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2}=\frac{\partial^2 u(x,t)}{\partial x^2} \tag{1}$$

where u is the pressure and c is velocity, the explicit finite difference scheme for fourth-order space and second-order time accuracy is,

$$u(x_m,t_{n+1})=2u(x_m,t_n)-u(x_m,t_{n-1}) + \frac{c^2\Delta t^2}{12*\Delta x^2}\begin{bmatrix} -u(x_{m+2},t_n)+16u(x_{m+1},t_n)-30u(x_m,t_n) \\ +16u(x_{m-1},t_n)-u(x_{m-2},t_n) \end{bmatrix} \tag{2}$$

To apply the ODD method to Eq. (2), the domain Ω is divided into two subdomains Ω1 and Ω2 (Fig. 2). The amount of overlap between these two subdomains can be determined as follows. The grid points ... , m-2, m-1, m belong to Ω1 and the grid points m+1, m+2, m+3, ... belong to Ω2. Assume that the pressures $u(x_i,t_n)$ and $u(x_i,t_{n-1})$ at time step n-1 and n are known values. In order to get $u(x_m,t_{n+1})$ in Ω1, two previous time step values are needed from the grid points m-2 through m+2. If Ω1 is overlapped to cover the grid points m+1 and m+2, then $u(x_m,t_{n+1})$ can be solved within Ω1.

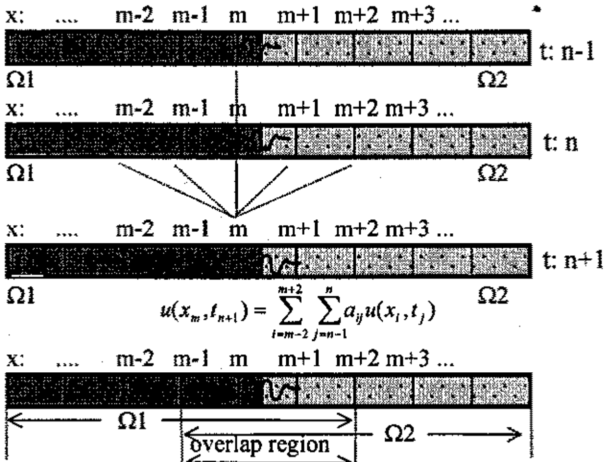

Fig. 2 Overlap grid scheme for 1-D FD method

The grid points m+1 and m+2 which originally only belonged to Ω2 now also belong to Ω1. That is, these points form the overlap region. $u(x_m,t_{n+1})$ can be calculated from Ω1 just as in the standard FD. Similarly, $u(x_{m+1},t_{n+1})$ can be calculated at the grid point m+1 in Ω2 using the values from grid points m-1 and m+3. The total overlap region now spans the grid points from m-1 through m+2. As can be seen in this analysis, the overlap region only needs four grid points (Fig. 2) to get exactly the same results as the standard FD method applied to total domain Ω. More grid points in the overlap region would give same results.

It should be noted that the results at grid points m+1, m+2 in Ω1 and m-1, m in Ω1 near the artificial boundaries are not used and only the uncontaminated results on the inside region are used to form the solution at time step n+1; Therefore, the artificial boundaries have no adverse effects in the ODD method.

ODD for 1-D Fourier pseudospectral (PS) method

The Fourier PS method has been successfully developed in recent years for acoustic and elastic wave propagation (Gazdag, 1981; Kosloff and Baysal, 1982; Kosloff et al., 1990; Furumura and Takenaka 1995). The ODD method can also be applied to the Fourier PS method. The PS method solution of Eq. (1) is

$$u(x_m,t_{n+1}) = +2u(x_m,t_n) - u(x_m,t_{n-1}) + c^2\Delta t^2\left[\frac{1}{N}\sum_{l=0}^{N}(-k_l^2) * \tilde{u}(k_l,t_n)e^{ik_lx_m}\right] \quad (3)$$

where $k_l = 2\pi l/(N*\Delta x)$, $\Delta x = x_m - x_{m-1}$ and the $\tilde{u}(k_l,t_n)$ are the Fourier transform of $u(x_m,t_n)$

$$\tilde{u}(k_l,t_n) = \sum_{m=0}^{N} u(x_m,t_n)e^{-ik_lx_m} \quad (4)$$

Two problems which arise when applying the ODD method developed in the precious section for the standard FD method to Eq. (3) are the need to known the pressure u at all the grid points of the previous time step and the wraparound which occurs at boundaries. Both these problems can circumvented by applying a taper to the pressure field at the boundaries of each subdomain (Liao and McMechen, 1993). By tapering the pressure field to zero near the boundaries of all the subdomains, Fourier wraparound contributions are zero and the transfer of artificial waves is eliminated.

ODD for 1-D mixed methods

Because of the independence of the computing algorithm used in subdomains of the ODD method, the FD and PS methods can be easily coupled together. In order to get smooth solutions across the subdomains, the different methods should have approximately the same accuracy.

ODD METHOD FOR 2-D PROBLEMS

The 1-D ODD methodology can be directly applied to 2-D problems by overlapping in two dimensions (Fig. 3).

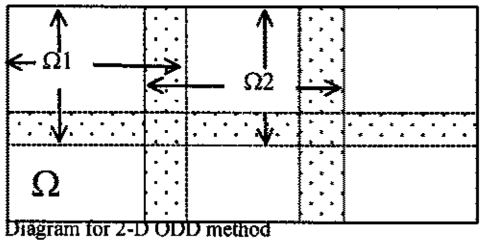

Fig. 3 Diagram for 2-D ODD method

The following table gives the minimum numbers of overlap grids for FD methods depending on their equations and approximations for different problems.

FD Methods	2nd order acoustic (const ρ)	4th order acoustic (const ρ)	2nd order acoustic or elastic	4th acoustic or elastic
Min overlap gird points	2	4	4	8

For the Fourier pseudospectral method, the width of overlap regions should be about one wavelength .

EXAMPLES

The finite difference method and pseudospetral method have been widely studied (Alford et al., 1974, Kelly et al., 1976, Dablain 1986, Gazdag, 1981, Kosloff and Baysal, 1982; Fornberg, 1987, 1996; Kosloff et al., 1990). The solutions obtained by using ODD FD and PS methods are compared with the results by using standard FD and PS methods.

The physical model for the 1-D comparative study is illustrated in Fig. 4. A Ricker wave with the central frequency of 25 Hz and wavelength of 120 m is used as the source wavelet. The total model is 2560 m long. The velocity is 3000 m/s. The distance between the source and receiver is 2200 m (approximately 18 wavelengths).

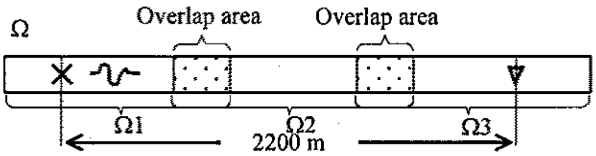

Fig. 4 1-D physical model for ODD tests

The standard 4th order FD (FD4), PS, ODD PS, ODD FD4, and mixed ODD PS-FD4 (PS in Ω, FD in Ω1 and Ω2) methods are applied to the model. The ODD methods only calculate the subdomains with wave energy.

The waveforms from different methods are showed in Fig. 5. The results match very well. The relative differences are less than 0.02% between the ODD and standard methods.

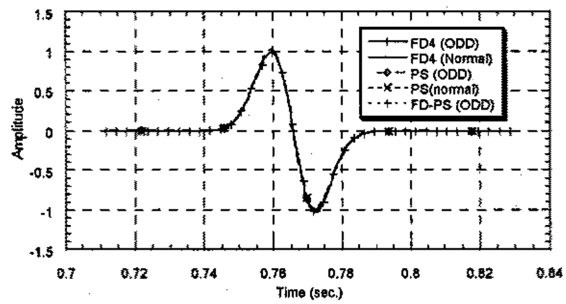

Fig. 5 Waveforms for 1-D model using different methods

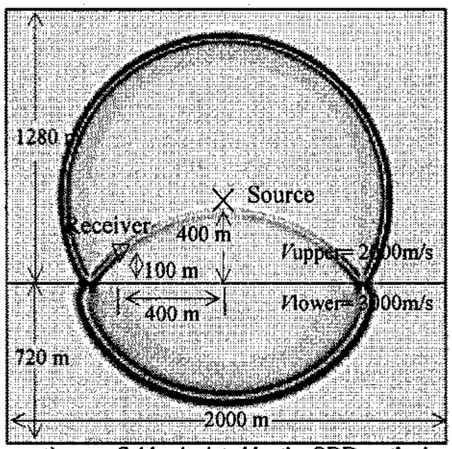

Fig. 6 Acoustic wavefield calculated by the ODD method

The 2-D acoustic model and a snapshot of acoustic wavefield calculated at time 0.4 sec with the ODD PS-FD4 method (25 subdomains) are shown in Fig. 6.

The waveforms calculated by different methods for the 2-D acoustic model are showed in Fig. 7. They all match very well. The relative differences are less than 0.1% between the ODD and standard methods. However, the ODD FD4 method only takes about the half computing time of the standard FD4 method and ODD PS method takes one third of the computing time of the standard PS method.

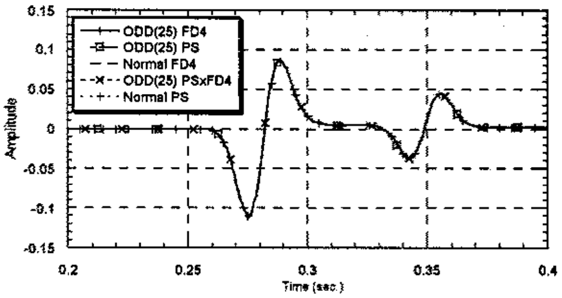

Fig. 7 Waveforms from different ODD methods for 2-D acoustic model

Fig. 8 shows the elastic model and a snapshot at time 0.4 sec of the vertical component of displacement calculated from the ODD PS-FD4 (25 subdomains) method with a horizontally polarized source.

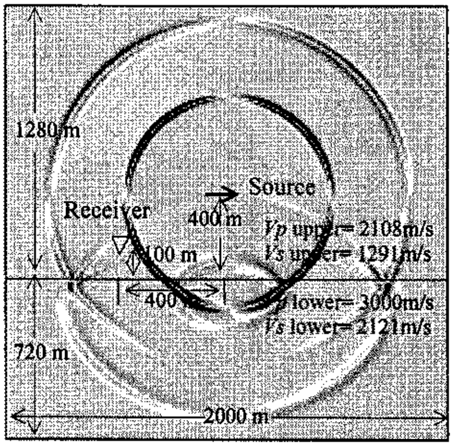

Fig. 8 Horizontal displacements calculated by the ODD method

CONCLUSION

This paper presents a computationally efficient method for simulating waves in large domains. The method uses an overlapping domain decomposition to divide a large computational domain into a series of smaller subdomains. Different numerical algorithms for computing wave propagation can be independently used in each subdomain. The method not only saves computing time by turning off computation in inactive subdomains but also gives the almost same accuracy as standard methods. The method is based on simple physics and can be easily implemented. This method has been successfully applied to the finite difference method and pseudospectral method and a combination of these methods for acoustic and elastic problems. Future work will apply this method to three dimensional problems.

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